GEOREFERENCING APPLYING WEIGHTING COEFFICIENTS FOR LEAST SQUARES ADJUSTMENT

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ABSTRACT

The article describes one of the problematic aspects of georeferencing old maps. It is the issue of weighting coefficients for ground control points. A certain number of ground control points always enters the transformation calculation. If global transformation methods are used, the adjustment applying the method of least squares is made in the case of a redundant number of ground control points available. The redundant number of ground control points always guarantees the check of the selection of these points. In the case that the standard positional deviation differs by an order, this point is suspicious and it should be eliminated from the calculation after the verification. The remaining ground control points vary by their different standard positional deviation. Ground control points should preferably be evenly distributed throughout the overall map drawing. The points are chosen on clearly identifiable objects on both the local and the reference map. The choice is always easier in urban areas. In rural areas, it mostly falls on road crossings, changes in the vegetation cover on land parcels or the confluence of rivers. These ground control points tend to have worse accuracy. The introduction of weighting coefficients is only suitable for global transformation methods. A source code has been created in the Matlab programme, which first calculates the classical affine or the second order polynomial transformation. At each point, the coordinate and positional difference from the coordinates identified during the collection of ground control points is calculated. Weighting coefficients are selected in two ways. The first way to determine the weighting coefficient is by the inverse absolute value of this coordinate difference. The other option for both coordinates of one control point is the weighting coefficient which equals the inverse of the positional difference. Georeferencing applying weighting coefficients is more appropriate for the result as the map is placed in the reference coordinate system with an emphasis on the ground control point accuracy. The points with a smaller coordinate/positional difference have a greater effect on the calculation, while points with a greater coordinate/positional differences affect the calculation to a lesser extent.

Keywords: georeferencing, transformation, old maps, weighting coefficients.

PROBLEMATIC ASPECTS IN GEOREFERENCING

The georeferencing of old maps is a process consisting of a series of individual acts. Not much emphasis is often put on georeferencing during the map digitising process, and georeferencing itself is affected with numerous errors. Frequent errors arising in georeferencing are pointed out by Baiocchi and Lelo [3]. The process of georeferencing old maps is described by Podobnikar [9] and Molnar, Podobnikar and Timar [10].

If we want to georeference an old map, first we must scan it. Scanning is performed on large-format calibrated scanners. A map is usually scanned with a resolution of 400 DPI. Higher resolutions than that mostly only increase the volume of data. The risk associated with the choice of a lower resolution, on the contrary, is losing map details. In the case of a resolution of 400 DPI, the size of one pixel is approximately 0.06 mm, and with regard to the minimum width of a line on the map of 0.1 mm, this resolution is sufficient.

The following step is to locate the scanned map into a coordinate reference system. This step is called georeferencing. The georeferencing of maps is mentioned by Zlinszky and Molnar [11]. Ground control points are identified on the scanned map for which coordinates in the local map system and in the respective coordinate reference system are available. It is of utmost importance in georeferencing to read the coordinates of ground control points from the same, or a very similar cartographic projection. The original scanned map is always made in some cartographic projection, and the coordinates identified on the background map should be in the same (or a very similar) cartographic projection. Cartographic projections with respect to georeferencing were studied in more detail by Havlíček and Cajthaml [7], Bayer [1], [2] and Boutoura and Livieratos [4].

After getting information on a map and determining ground control points (Havlicek and Cajthaml [8]), the next important step is to choose the type of transformation method and the very type of transformation. There are two types of transformation methods used in practice. The first group are global transformation methods by means of which one transformation key is calculated from all ground control points using the Least Squares Method. This key, in turn, is applied to the whole area covered by the map, and, therefore, ground control points do not have their own position in the final result as some deviations arise at these points in the case of redundant ground control points. The second group of used transformation methods are local methods. A unique transformation key is calculated for each point on the map. Ground control points have their own position in the final result of this method. A major drawback is that in the case of a wrongly determined ground control point this point cannot be identified, and a relatively large map distortion arises in its vicinity.

The following step usually involves saving information on georeferencing. Currently, there are three saving options used. The first is creating a world file where information is stored in an auxiliary file to a raster file by means of six numbers – the pixel size in the x and y axis, the rotation about the x and y axes, and the coordinates of the upper left corner of the raster. The drawback of this type of saving is that only global transformation methods, namely identity, similarity and affinity transformation, can be saved within these six values. If higher order polynomial transformations or local transformation methods are used, the results can no longer be saved in six unknown parameters. Another option is to save information in an XML file where data on the coordinate reference system, ground control points, transformation type, etc. are also saved. This type of saving is the best considering the future use. Unfortunately, there is currently no single XML available for different geographical information programmes, and e.g. the ESRI Company uses its own AUX.XML file. The third option is resampling the georeferenced raster. A new raster arises in this case which always has a worse resolution than the original as the pixel values are derived from the surrounding pixels using a variety of methods. The drawback of this option is the loss of information on ground control points. For more details on the possibilities of saving the results of georeferencing, consult publications by Cajthaml [5], [6].

Other problematic aspects include the georeferencing of a map series consisting of more map sheets and the introduction of weighting coefficients for ground control points, which will be described in more detail in this article.

ADJUSTMENT CALCULATION USING WEIGHTING COEFFICIENTS

During the calculation of global transformation methods in georeferencing, the adjustment is made in the case of a redundant number of ground control points available. As a consequence, the position of adjusted ground control points does not correspond to the precise position which was read during their saving. The coordinate and, subsequently, the positional difference can be additionally calculated at adjusted ground control points.

Ground control points are identified on various objects. On medium- and small-scale maps, settlements are usually only marked by a point symbol. Settlements are classified by type (e.g. town, township, village with a church, village, farmstead). On large-scale maps, ground control points are chosen on buildings, road crossings, confluences of streams, changes in landscape types, etc. It is assumed that the more significant the map element (settlement, building) is, the more precisely this element is displayed on the map. This rule, however, does not apply invariably. In the case that the cartographer is not sure whether the ground control point sample can be subdivided and weighting coefficients can be assigned to individual types of ground control points, it is more advisable to use a general rule. Providing the assumption that the majority of ground control points were correctly identified is met, the positional or coordinate difference should be minimum at correctly identified points and maximum at erroneously identified points. The more precisely a ground control point is determined, the greater weighting coefficient should be assigned to this point and vice versa.

According to the above hypothesis, it was decided that weighting coefficients would be identified in two ways. In the first method, they will be identified as the inverse of the positional difference for both coordinates of ground control points. In the second method, they will be identified as the inverse of the absolute value of the coordinate difference for individual coordinates of ground control points.

First, a general adjustment is made using the least squares method, see formula 1. The matrix A is the matrix of partial derivatives of individual variables (6 coefficients for the affine transformation and 12 coefficients for the second order polynomial transformation), and the vector l is the vector of identified coordinates of ground control points.

$$x = (A^T \cdot A)^{-1}(A^T \cdot l) \qquad (1)$$

The resulting coordinates of ground control points are additionally calculated by means of the vector x. The difference in the identified and calculated coordinates of ground control points serves for the determination of the weighting coefficient, which equals the inverse of the absolute value of the coordinate difference for coordinate weighting coefficients and the inverse of the positional difference of a respective ground control point for positional weighting coefficients.

The adjustment using the least squares method is successively made again, using the weight matrix P – see formula 2. For coordinate weighting coefficients, these values are placed on the main diagonal of the matrix P. For positional weighting coefficients, these coefficients are placed twice after each other on the main diagonal of the matrix P.

$$x = (A^T \cdot P \cdot A)^{-1}(A^T \cdot P \cdot l) \quad (2)$$

The application of weighting coefficients was simulated by means of the source code in the MATLAB programme. The resulting positional deviations at ground control points were used to interpolate a regular grid which was graphically plotted in a chart.

APPLICATION OF WEIGHTING COEFFICIENTS

In applying weighting coefficients, the source code of the MATLAB programme was tested on several data samples. The source code was created for two types of transformations – the affine transformation and the second order polynomial transformation. Both the positional weighting coefficient for both coordinates of a ground control point and the coordinate weighting coefficient for individual coordinates was tested for both transformation types.

Figures 1 and 2 display the application of weighting coefficients (affine transformation and second order polynomial transformation) for one map sheet of the Emperor’s Print of the Stable Cadastre, which was georeferenced from the local system into the S-JTSK coordinate system. 18 ground control points were identified on the map drawing.

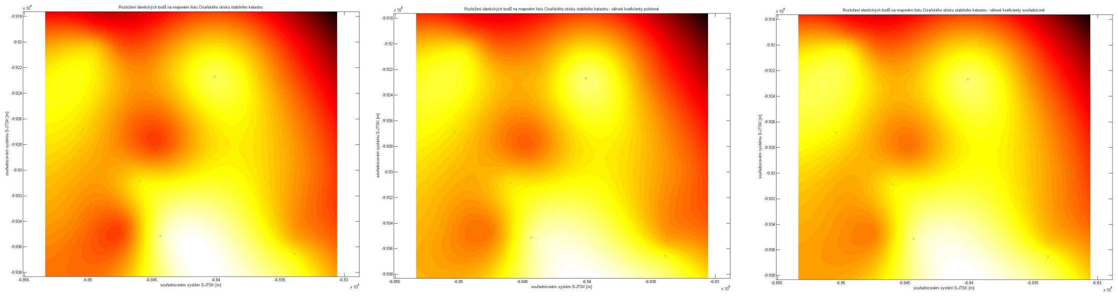

Figure 1: Affine transformation of the Emperor’s Print of the Stable Cadastre (without, with positional and with coordinate weighting coefficients)

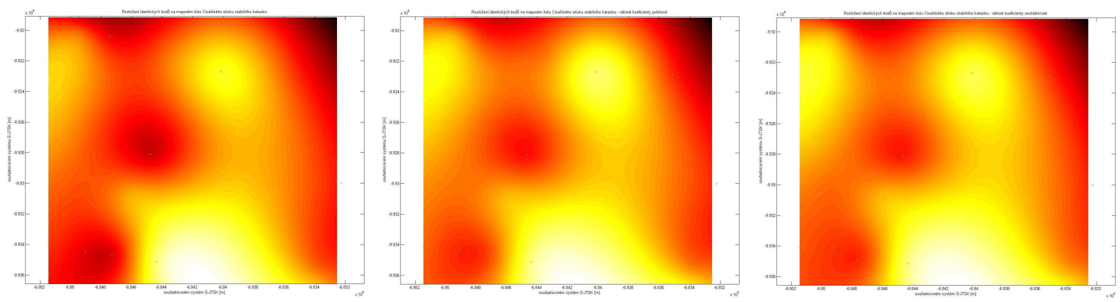

Figure 2: Second order polynomial transformation of the Emperor’s Print of the Stable Cadastre (without, with positional and with coordinate weighting coefficients)

The standard positional deviations of individual positional alignment the Emperor's Print of the Stable Cadastre are shown in Table 1.

Table 1: The standard positional deviations of the Emperor's Print of the Stable Cadastre

Transformation	Without weighting coefficients	With positional weighting coefficients	With coordinate weighting coefficients
Afinne	3,39 m	3,41 m	3,40 m
The second order polynomial	2,69 m	2,73 m	2,73 m

Figures 3 and 4 display the application of weighting coefficients (affine transformation and second order polynomial transformation) for towns without ramparts and townships on Müller's Map of Bohemia of 1721, which was georeferenced from the local system into the S-JTSK coordinate system. 319 ground control points were identified on the map drawing.

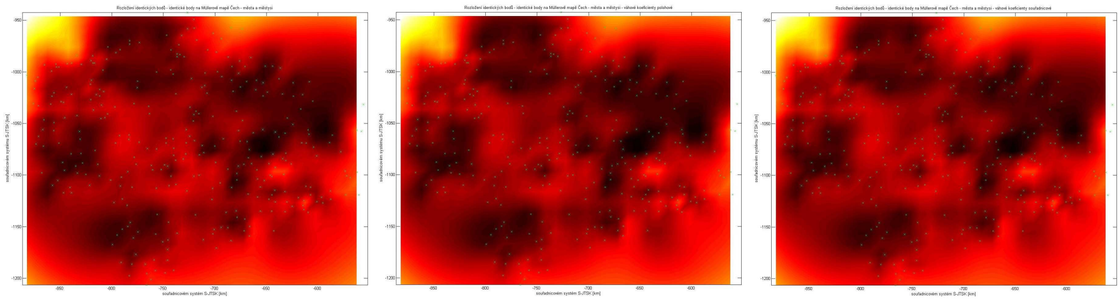

Figure 3: Affine transformation of the Müller's map of Bohemia (without, with positional and with coordinate weighting coefficients)

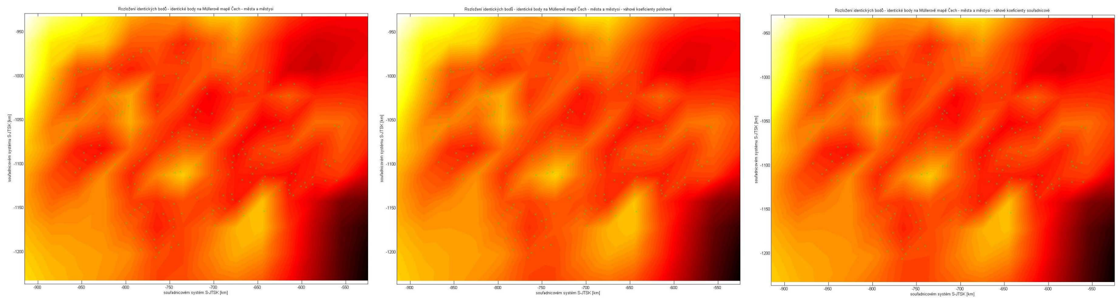

Figure 4: Second order polynomial transformation of the Müller's map of Bohemia (without, with positional and with coordinate weighting coefficients)

The standard positional deviations of individual positional alignment the Müller's map of Bohemia are shown in Table 1.

Table 2: The standard positional deviations of towns without ramparts and townships on the Müller's map of Bohemia

Transformation	Without weighting coefficients	With positional weighting coefficients	With coordinate weighting coefficients
Afinne	1,927 m	1,931 m	1,928 m
The second order polynomial	1,420 m	1,422 m	1,421 m

Comparing the coordinate difference it is evident that the points with a greater accuracy show a lower coordinate difference after the introduction of weighting coefficients. In the case of the introduction of the positional weighting coefficient, the magnitude of the coordinate difference of an originally small coordinate difference may even grow. This case occurs if one coordinate is significantly more precise than the other.

CONCLUSION

The article describes the adjustment calculation by means of the least squares method with weighting coefficients. It was found out that in the case of using the absolute value of the inverse of the coordinate difference at a ground control point as the weighting coefficient, the standard positional deflection of the calculation is nearly constant. For points with an originally lower coordinate difference at a ground control point, this difference is still smaller after the calculation. For points with an originally greater coordinate difference at a ground control point, on the contrary, this difference is still greater. While using the inverse of the positional difference as the weighting coefficient, the standard positional deflection hardly differs again. The result is very often the same as in the case of using coordinate weighting coefficients. A change only occurs at those ground control points which have a markedly different accuracy of their coordinates.

In mathematical terms, the application of weighting coefficients does not make the total transformation result more precise as the standard positional deviation is nearly constant. The graphic result, however, is more precise as the points with a greater accuracy have their resulting position closer to their identified coordinates after the introduction of weighting coefficients. In contrast, the points which were not so precisely identified have a greater coordinate difference from their identified position after the introduction of weighting coefficients.

For this reason, I would recommend introducing coordinate weighting coefficients into practice.

The calculation of weighting coefficients must be performed in an external programme as none of GIS programmes allows the application of weighting coefficients directly in the calculation. The application of the resulting coefficients to the least number of control points, which will be used for the transformation in the GIS programme, follows.

ACKNOWLEDGEMENTS

This project was supported from the NAKI - DG16P02H010 grant project "Czech Historical Atlas". The knowledge presented in this article may be applied for the georeferencing of historical background maps necessary for the creation of the Czech Historical Atlas.

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